Fluctuations in uncertainty, efficient financial constraints and firm dynamics

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Motivation

Key empirical observation:

Age rather than size of a firm is a determinant of the cyclical employment dynamics.

Look through the lens of **endogenously** frictional financial markets theory. Two ubiquitous features:

- relevance of past performance
- long-term nature of financial arrangements

Question:

To what extent aggregate fluctuations in micro uncertainty, propagated through financial frictions, account for aggregate employment dynamics and asymmetric employment patterns across various groups of firms?

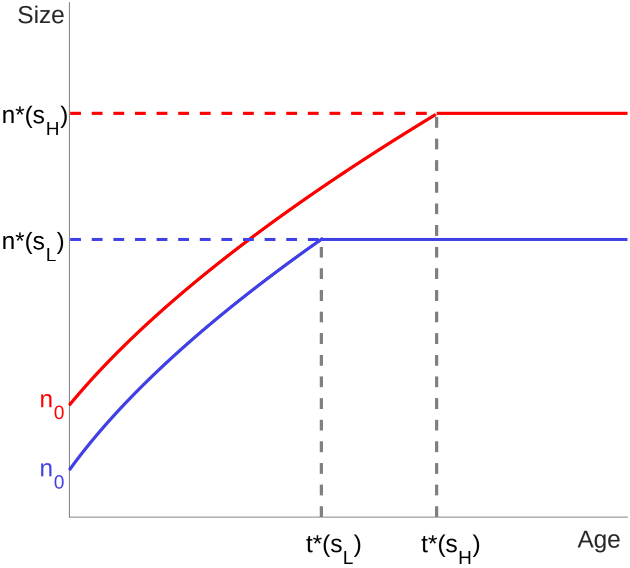
This paper

1. Long-term contracts
   - optimal arrangement between firm and intermediary
2. Endogenous financial friction
   - private information as an origin of the financing constraint
3. Micro uncertainty - real economic activity nexus
   - uncertainty as a determinant of financial friction severity

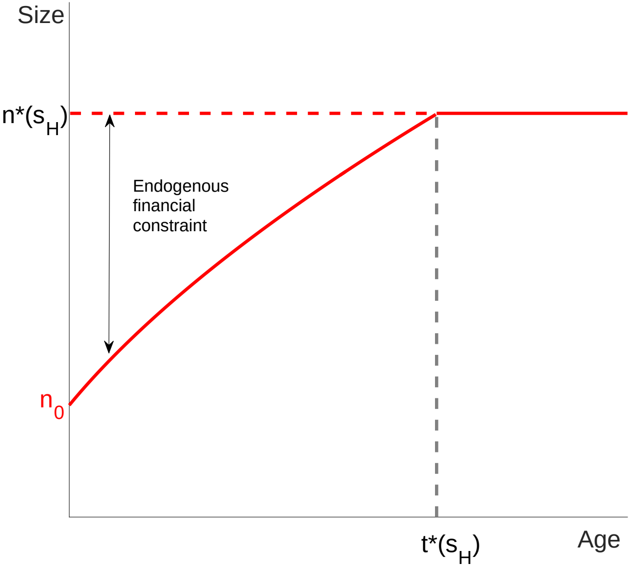
Related Literature

- Firm dynamics:
  Gertler, Gilchrist (1994), Christiano et al. (2008), Moscarini, Postel-Vinay (2012), Haltiwanger et. al. (2013)
- Private information in dynamic contracting:
  Thomas, Worall (1990), Clementi, Hopenhayn (2006), DeMarzo, Fishman (2007), Smith, Wang (2005), DiTella (2014), Verani (2014)
- Uncertainty shocks:
  Bloom (2009), Bloom et. al. (2012), Arellano et. al. (2012), Gilchrist (2014), Christiatno et. al. (2014)
- Financial frictions:
  Bernanke, Gertler (1989), Kiyotaki, Moore (2008), Jermann, Quadrini (2011), Shoudrieh, Zetlin-Jones (2012)

Overview of the mechanism



Overview of the mechanism



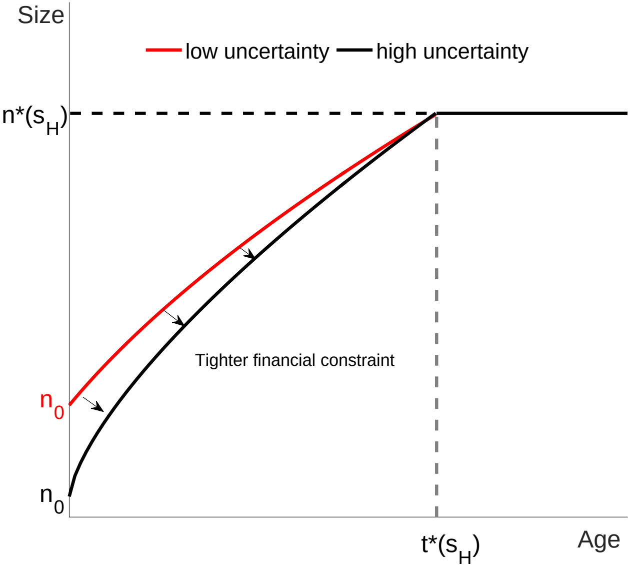
Economic Downturn in the Model Economy

- **Initial impulse**: aggregate shock to volatility of firm's idiosyncratic demand/productivity (mean preserving)
- **Propagation mechanism**:

$$\begin{array}{c}\text{larger dispersion in productivity realizations}\\ \downarrow \\ \text{more incentives to misreport and get extra consumption}\\ \downarrow \\ \text{spread out continuation values to separate types - costly}\\ \downarrow \\ \text{cost balanced by tightening of the financial constraint}\end{array}$$

- **Real effects**: fall of the demand for production inputs + general equilibrium effects

Economic Downturn in the Model Economy



FACTS

Small $\neq$ Young

	All ages	Young (0-5)	Old (6+)
All sizes	100.0	**42.3**	57.7
Small (1-99)	**98.0**	42.0	**56.0**
Large (100+)	2.0	0.3	1.7

Distribution of firms. Averages, 1982-2012. Source: BDS.

- Young firms mostly small, but small not necessarily young.

Employment shares

Cyclical employment fluctuations: Age matters

	All ages	Young (0-5)	Old (6+)
All sizes	1.47	**3.20**	1.25
Small (1-99)	1.31	2.37	**1.02**
Large (100+)	1.67	**7.66**	1.50

Standard deviations of logged, HP filtered employment (1982-2012).

- Young 2.6 times more volatile than old.
- Cyclicality declines with age and increases with size.

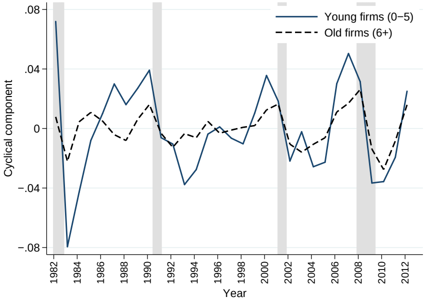
...but it is not due to entry

	All ages	Young (1-5)	Old (6+)
All sizes	1.45	**3.17**	1.25
Small (1-99)	1.29	2.34	**1.02**
Large (100+)	1.66	**7.58**	1.50

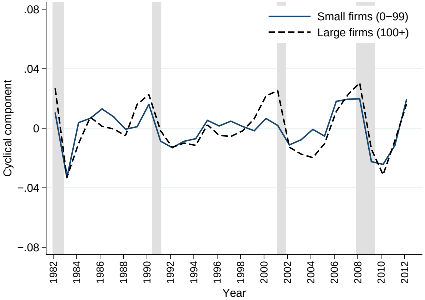
Standard deviations of logged, HP filtered employment (1982-2012).

- Movements in the number of startups do not matter.

Cyclical component of employment and recessions



(a) Young vs. old ($\sigma_y/\sigma_o = 2.61$)



(b) Small vs. large ($\sigma_s/\sigma_l = 0.75$)

Notes: Shaded areas are NBER recessions. Employment series are logged and HP filtered with parameter $\lambda = 6.25$. Source: Own calculations, BDS 1982-2012.

- All groups positively correlated (0.80 to 0.94), also with cyclical component of GDP (0.45 to 0.63).

MODEL

Environment

- Time is discrete, lasts forever and is indexed by $t = 0, 1, ...$
- Agents: a large number of workers, a large number of firms (entrepreneurs) and financial intermediaries.
- A single consumption good in the economy.
- Firm's project type $s \in S$. Idiosyncratic stochastic productivity $\theta \in \Theta$.
- Aggregate shock: variance σ_θ of productivity shock is moving.

Timing

The timing of the events within a period:

1. Firms enter the period with predetermined capital and labor inputs.
2. Aggregate shock σ_θ and idiosyncratic shock θ are realized. θ is **private information**.
3. Production takes place. Consumption (privately observed) and payments take place.
4. Firms learn whether they survive (with prob. ζ). New firms are born and draw type s from Γ.
5. Financial intermediaries provide capital to the firms. Labor is hired.

Entrepreneurs: Preferences

- Firm is associated with an entrepreneur.
- A time-invariant probability $\zeta < 1$ of surviving into the next period. Initial net-worth: M.
- Firm that starts operating in period j, values $\{c_t\}_{t=j}^{\infty}$ through the lens of the entrepreneur's preferences, i.e.

$$\sum_{t=j}^{\infty} \sum_{h_s^t} (\beta\zeta)^{t-j} \Pr\left(h_s^t\right) U\left(c\left(h_s^t\right)\right)$$

where $h_s^t = (\theta_{s,j}, \sigma_{\theta j}, \theta_{s,j+1}, \sigma_{\theta,j+1}, ..., \theta_{s,t}, \sigma_{\theta,t})$.

- Entrepreneur is risk averse. Firm's age is $(t-j)$.

Entrepreneurs: Technology

- Access to a decreasing returns to scale technology: $f(\theta, k, n) = Avk^{\alpha}n^{\gamma}$, where

$$v = s^{1-\gamma} + \frac{\sigma\theta}{\pi(\theta)}$$

with $\theta \in \{-1, 1\}$ and $\sigma \in \{\sigma_L, \sigma_H\}$.

- Type s determines expected productivity i.e. $\mathbb{E}(v) = s^{1-\gamma}$.
- Labor choice solves

$$\max_n A\mathbb{E}[v]k^{\alpha}n^{\gamma}$$

and the optimal labor allocation by $n^*(k)$.

- Denote

$$F(\theta, k) \equiv f(\theta, k, n^*(k)) + (1-\delta)k$$

Financial intermediaries and workers

Financial intermediaries

- risk neutral, value stream of consumption good with discount factor $\frac{1}{1+r}$
- provide funds to firms in the exchange for payments
- free entry: representative financial intermediary

Workers solve static problem

$$\max_{C^w, H} U\left(C^w, H\right) \quad \text{s.t. } C^w = wH$$

where $U(C,H) = \frac{1}{1-\rho}\left(C - \psi \frac{H^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right)^{1-\rho}$.

Feasible, incentive compatible long-term contract

- Firm's state variable: v_s promised utility for type s.
- A **dynamic contract** is: $\{k, m(\theta,\sigma'), c(\theta,\sigma'), v'(\theta,\sigma')\}$ where k is capital, $m(\theta,\sigma')$ is repayment, $c(\theta,\sigma')$ is entrepreneur's consumption and $v'(\theta,\sigma')$ is continuation value.
- Feasibility constraint

$$c\left(\theta,\sigma'\right) + m\left(\theta,\sigma'\right) = F\left(k,\theta,\sigma'\right) \quad \forall\theta,\sigma'$$

- Incentive compatibility constraint

$$U\left(c\left(\theta,\sigma'\right)\right) + \zeta v'\left(\theta,\sigma'\right) \geq U\left(F\left(k,\hat{\theta},\sigma'\right) - F\left(k,\theta,\sigma'\right) + c\left(\hat{\theta},\sigma'\right)\right) + \zeta v'\left(\hat{\theta},\sigma'\right) \quad \forall\hat{\theta},\theta,\sigma'$$

- Promise keeping constraint

$$v_s = \beta\sum_{\theta}\sum_{\sigma'}\left[U\left(c\left(\theta,\sigma'\right)\right) + \zeta v'\left(\theta,\sigma'\right)\right]\pi\left(\theta\right)\pi\left(\sigma'|\sigma\right)$$

Optimal long-term contract - recursive formulation

Optimal financial contract solves

$$B_s\left(v_s,\sigma,\mu\right)=\max_{k,c,m,v'}\left\{-k+\mathbb{E}\left[\frac{m\left(\theta,\sigma'\right)+\zeta B_s\left(v'\left(\theta,\sigma',\mu'\right)\right)}{1+r}\right]\right\}$$

subject to

$$v_s=\beta\mathbb{E}\left[U\left(c\left(\theta,\sigma'\right)\right)+\zeta v'\left(\theta,\sigma'\right)\right]$$

$$U\left(c\left(\theta,\sigma'\right)\right)+\zeta v'\left(\theta,\sigma'\right)\geq U\left(F\left(\theta,\sigma',l\right)-F\left(\widehat{\theta},\sigma',l\right)+c\left(\widehat{\theta},\sigma'\right)\right)+\zeta v'\left(\theta,\sigma'\right)\qquad\forall\hat{\theta},\theta,\sigma'$$

$$c\left(\theta,\sigma'\right)+m\left(\theta,\sigma'\right)=F\left(\theta,\sigma',l\right)\qquad\forall\theta,\sigma'$$

$$\mu'=\Gamma_\mu\left(\mu,\sigma\right)$$

Technical details

Optimal long-term contract - recursive formulation

Optimal financial contract solves

$$B_s\left(v_s,\sigma,\mu\right)=\max_{k,c,v'}\left\{-k+\mathbb{E}\left[\frac{F\left(\theta,\sigma',l\right)-c\left(\theta,\sigma'\right)+\zeta B_s\left(v'\left(\theta,\sigma',\mu'\right)\right)}{1+r}\right]\right\}$$

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$$\forall\hat{\theta},\theta,\sigma'$$

$$\mu'=\Gamma_\mu\left(\mu,\sigma\right)$$

Technical details

Aggregation

- Perfect competition in the financial market pins down v_s^0

$$B_s\left(v_s^0\right) = 0$$

- Aggregate capital, labor and payments are

$$\begin{aligned} K &= \sum_{s\in S}\Gamma_s \int_V k\left(v_s,\sigma\right) d\mu_s\left(v_s\right), N = \sum_{s\in S}\Gamma_s \int_V n\left(v_s,\sigma\right) d\mu_s\left(v_s\right) \\ P &= \sum_{s\in S}\Gamma_s \int_V \pi\left(\theta\right) m\left(v_s,\theta,\sigma\right) d\mu_s\left(v_s\right) \end{aligned}$$

- Asset holdings of the financial intermediary

$$A' = \left(1+r\right) A + \left(P-K\right)$$

- Market clearings for labor and consumption good.

Stationary recursive equilibrium

A **recursive competitive equilibrium** consists of: (i) an allocation of the household $\{C^w, H\}$ (ii) a contract policy $\{k, m(\theta, \sigma'), c(\theta, \sigma')\}_{s \in S}$ (iii) an allocation of labor $\{n(v_s), k(v_s)\}_{s \in S}$ (iv) price $\{w\}$ (v) initial promised utility value v_s^0 (vi) the measure μ over the space of promised utility, such that :

1. Given $\{w\}$, an allocation $\{C^w, H\}$ solves the problem of the workers.
2. Contract policy solves the optimal contracting problem.
3. Given $\{w\}$, an allocation $\{n\}_{s \in S}$ solves the problem of the firm.
4. Labor market clears: $N = H$.
5. The initial promised utility v_s^0 satisfies zero profit condition.
6. The evolution of the joint distribution of θ, v is consistent. That is, $\Gamma_\mu(\sigma, \mu)$ is generated by $V(v_s, \sigma)$ and the exogenous stochastic evolution of σ.

Full info vs private info: key trade-offs

- A full information, efficient level of capital, k^*, is determined by
$$\frac{1}{1+r}\mathbb{E}\left[F'\left(k^*,\theta\right)\right]=1.$$
- The presence of the private information induces trade-offs between:
  - production efficiency
  - insurance provision
  - providing proper intertemporal incentives to induce truth-telling
- Trade-offs manifest as wedge:
$$\frac{1}{1+r}\mathbb{E}\left[F'\left(k,\theta,\sigma_\theta\right)\right]=1+\underbrace{\boldsymbol{\Omega}\left(\cdot,\sigma_\theta\right)}_{>0}.$$

Private information: contract properties

Proposition 1

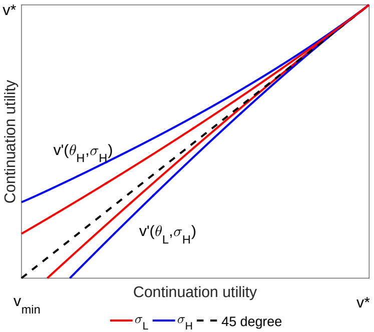
For all s, σ, σ' a contract policy is such that:

(i) *The contract policy is dynamic:* $\forall v \in [v_{\min}, v_{\max}]$, $m(\theta_i, \sigma') > m(\theta_j, \sigma')$, $c(\theta_i, \sigma') > c(\theta_j, \sigma')$, *and* $v'(\theta_i, \sigma') > v'(\theta_j, \sigma')$ *for* $\theta_i > \theta_j$.

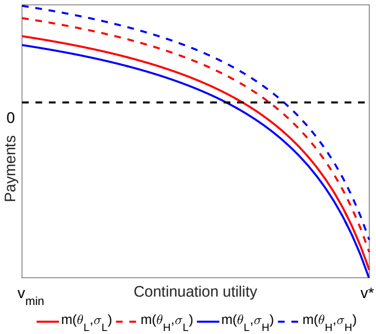
(ii) *There are distortions in financing. There exists* $v^* \in [v_{\min}, v_{\max}]$ *such that* $k(v, \sigma) < k^*$ *for all* $v \in [v_{\min}, v^*]$ *and* $k(v, \sigma) = k^*$ *for all* $v \in [v^*, v_{\max}]$.

- To maintain incentives continuation utilities are spread out. Imperfect insurance.
- Informational friction generates an **endogenous financial constraint**, i.e. $(k^* - k(v, \sigma)) > 0$ for all $v \in [v_{\min}, v^*]$.

Contract policy functions and role of uncertainty

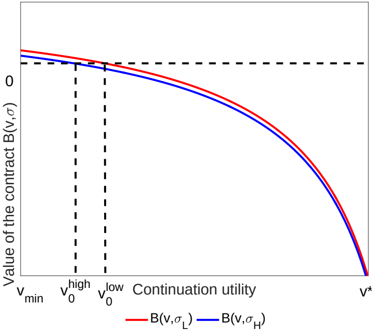


(a) Continuation values

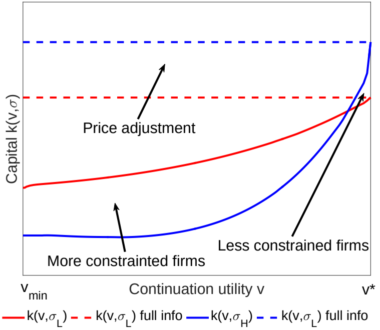


(b) Payments

Contract policy functions and the role of uncertainty



(a) Value function



(b) Payments

- Distribution of firms over constrained and unconstrained region matters for employment response.
- Discipline: match age/size distribution of firms in the US

Calibration: preferences and technology

Table: Exogenously Determined Parameters of the Baseline Economy

Parameter	Value
Inverse of IES, ρ	2.0
Frisch elasticity of labor, ν	2.0
Share of capital, α	0.25
Share of labor, η	0.50

Table: Preference and Technology Parameters and Associated Targets

Parameter	Value	Target	Value	Model
A	0.97	Aggregate output	1.0	1.0
δ	0.03	Investment-to-Output ratio	0.25	0.25
β	0.99	Interest rate r	0.01	0.01
ζ	0.95	Share of young firms in total	0.41	0.41
ψ	2.2	Average hours worked	0.30	0.30

Calibration: size/age distribution and uncertainty

Parameter	Value	Target	Value	Model
s_2/s_1	1.37	Employment share of large	62.3	58.6
Γ_1	0.98	Firm's share of group 1	0.98	0.98
Γ_2	0.02	Firm's share of group 2	0.02	0.02
M	0.45	Employment share of young	0.16	0.19

Notes: Group 1 has 1-99 employees; Group 2 has 100+ employees. Young firms are less 0-5 and old 6+. Source: BDS

Parameter		Value
Quarterly standard deviation of micro productivity shocks, %	σ_L	5.1
Micro volatility increase in high uncertainty state	σ_H/σ_L	4.1
Quarterly transition probability from low to high uncertainty, %	$\pi\left(\sigma_H \mid \sigma_L\right)$	2.6
Quarterly probability of remaining in high uncertainty, %	$\pi\left(\sigma_H \mid \sigma_H\right)$	0.943

Source: Bloom et. al. (2014)

Business cycle statistics

	Data		Model GE		Model PE	
	$\sigma(x)$	$\sigma(x)/\sigma(y)$	$\sigma(x)$	$\sigma(x)/\sigma(y)$	$\sigma(x)$	$\sigma(x)/\sigma(y)$
Output	1.2	1.0	0.7	1.0	0.9	1.0
Investment	6.2	5.2	3.9	5.6	4.3	4.7
Consumption	0.9	0.7	0.5	0.7	0.6	0.7
Labor	1.8	1.5	0.9	1.2	1.1	1.2

Notes: Data based on quarterly series between 1982 and 2012. Output is real gross domestic product, investment is real gross private domestic investment, consumption is real personal consumption expenditures, and hours is total nonfarm business sector hours. Right panel contains business cycle statistics from unconditional simulation of the model. All series are HP-filtered with smoothing parameter 1600, in logs expressed as percentages.

- Model accounts for **54%** of the aggregate employment volatility. GE effect dampens the overall volatility due to falling wage rate.

Asymmetric response across firms

Table: Effect of uncertainty shock on employment (std dev)

Age/Size	Data	Baseline model
Total	1.47	0.95
Young (0-5)	3.20	1.68
Old (6+)	1.25	0.81
Small (1-99)	1.31	0.91
Large (100+)	1.67	0.97

Notes: Annual data 1982-2012. All series are HP-filtered with smoothing parameter 6.25, in logs expressed as percentages.

- The ratio of std of young to old in the model is **2.1** vs. **2.6** in the data.

Conclusions

Two takeaways from the paper:

(1) Aggregate fluctuations partially rationalized as the response of constrained efficient equilibrium to aggregate shocks to micro uncertainty.

(2) Model offers and quantifies a mechanism to account for asymmetric cyclical employment patterns for different groups of firms.

Strengthen propagation mechanism:

- intermediate goods
- downward rigidity of wages

Distribution of employment

	All ages	Young (0-5)	Old (6+)
All sizes	100.0	**16.0**	84.0
Small (1-99)	37.7	12.6	25.1
Large (100+)	62.3	3.4	**58.9**

Shares of total employment. Averages, 1982-2012. Source: BDS.

- Employment concentrated among large old and large.

Firm distribution

Technical details

Assumption 1

Let $C : [U(0), U(\infty)] \to \mathbb{R}$ and $C = U^{-1}$, $H = F^{-1}$ and $u(\theta) = U(c(\theta))$ and $\underline{u} = U\left(\left(\theta_i + \theta_j\right) F(l) + C(U(c_j))\right)$ and . Define a function

$$G\left(\underline{u}, u\right) = -H\left(\frac{C(\underline{u}) - C(u(\theta_i))}{\theta_i - \theta_j}\right) + \frac{C(\underline{u}) - C(u(\theta_j))}{\theta_i - \theta_j}$$

where $\theta_i > \theta_j$. G is concave.

Lemma 1

(i) Under Assumption 1 for every $s \in S$ value function $B_s : [v_{\min}, v_{\max}] \to \mathbb{R}$ is strictly concave and maximizers $v'\left(\theta_s\right), m\left(\theta_s\right)$, $l\left(v_s\right), c\left(\theta_s\right)$ are continuous, singled-valued functions.
(ii) The value function B_s is differentiable.

Recursive formulation

Assumption 2

Assume $U(c) = \frac{c^{1-\rho}}{1-\rho}$ *with* $\rho > 1$*. Let* $N = 2$ *with* $\Theta = \{\theta_{sL}, \theta_{sH}\}$ *for all* $s \in S$ *with* $\pi_L = 1 - \pi_H$ *and let*

$$\theta_{sH} = \left(\overline{\theta_s} + \frac{\sigma}{\pi_H}\right)^{\frac{1}{1-\gamma}}, \quad \theta_{sL} = \left(\overline{\theta_s} - \frac{\sigma}{\pi_L}\right)^{\frac{1}{1-\gamma}}$$

implying $\mathbb{E}\left[\theta_s^{1-\gamma}\right] = \overline{\theta_s}$ *and* $std\left(\theta_s^{1-\gamma}\right) = \frac{\sigma}{\sqrt{\pi_L \pi_H}}$.

◂ Size slide

Related Literature

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- Financial frictions:
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◂ Main

Dynamic, feasible contract

- A firm is offered a financial contract contingent on all public information.
- Perfect commitment on both sides.

Definition 1

A **dynamic contract** *is* $\mathbf{x}_s \equiv \left\{ l\left(\theta_s^{t-1}\right), c\left(\theta_s^t\right), m\left(\theta_s^t\right) \right\}_{t=j}^{\infty}$ *for each* $s \in S$, *where* $l : \Theta_s^{t-1} \to \mathbb{R}_+$ *is lending,* $c : \Theta_s^t \to \mathbb{R}_+$ *is consumption and* $m : \Theta_s^t \to \mathbb{R}$ *is transfer to the intermediary.*

Definition 2

A dynamic contract $\mathbf{x}_s$ *is* **feasible** *if* $\forall t \geq j$ *and* $\forall \theta_s^{t-1} \in \Theta_s^{t-1}$, $\forall \theta_{st}$

$$c\left(\theta_s^t\right) + m\left(\theta_s^t\right) \leq \theta_{st}^{1-\gamma} F\left(l\left(\theta_s^{t-1}\right)\right) \tag{BC}$$

Information

- Feasibility requires for every $s \in S$ and $\forall \theta_s^{t-1} \in \Theta_s^{t-1}$, $\forall \theta_{st}$

$$c\left(\theta_s^t\right) + m\left(\theta_s^t\right) \leq \theta_{st}^{1-\gamma} F\left(l\left(\theta_s^{t-1}\right)\right)$$

- Observable for the intermediary:
  - permanent technology draw s
  - loan $l\left(\theta_s^{t-1}\right)$
  - payments $m\left(\theta_s^t\right)$
- Unobservable for the intermediary:
  - shock realization $\theta_{st}^{1-\gamma}$
  - consumption $c\left(\theta_s^t\right)$

Information

- Feasibility requires for every $s \in S$ and $\forall \theta_s^{t-1} \in \Theta_s^{t-1}$, $\forall \theta_{st}$

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- Observable for the intermediary:
  - permanent technology draw s
  - loan $l\left(\theta_s^{t-1}\right)$
  - payments $m\left(\theta_s^t\right)$
- **Unobservable** for the intermediary:
  - shock realization $\theta_{st}^{1-\gamma}$
  - consumption $c\left(\theta_s^t\right)$

Incentive compatible contract

- The continuation utility for the entrepreneur associated with the contract $\mathbf{x}_s$ after history θ_s^t

$$v\left(\theta_s^t\right) \equiv \sum_{n=1}^{\infty} \sum_{\theta_s^{t+n}} (\beta\zeta)^{n-1} \Pr\left(\theta_s^{t+n} | \theta_s^t\right) U\left(c\left(\theta_s^{t+n}\right)\right)$$

Definition 3

A dynamic contract $\mathbf{x}_s$ is **incentive compatible** *if it satisfies the incentive compatibility constraint $\forall t \geq j$, $\forall \theta_s^t \in \Theta_s^t, \forall \theta_{st}, \theta'$:*

$$U\left(c\left(\theta_s^t\right)\right) + \beta\zeta v'\left(\theta_s^t\right) \geq \tag{IC}$$
$$U\left(\left(\theta_{st}^{1-\gamma} - \theta'^{1-\gamma}\right) F\left(l\left(\theta_s^{t-1}\right)\right) + c\left(\theta_s^{t-1}, \theta'\right)\right) + \beta\zeta v'\left(\theta_s^{t-1}, \theta'\right)$$

Optimal contract

- In period j contract delivers the initial promised utility v_s^0

$$\sum_{t=j}^{\infty} \sum_{\theta_s^t} (\beta\zeta)^t \Pr\left(\theta_s^t\right) U\left(c\left(\theta_s^t\right)\right) \geq v_s^0 \tag{PC}$$

Definition 4

A feasible, incentive compatible contract $\mathbf{x_s}$ is **optimal** *if it solves the following problem*

$$J\left(v_s^0\right) = \max_{\mathbf{x}_s} \sum_{t=j}^{\infty} \sum_{\theta_s^t} \left(\frac{\zeta}{1+r}\right)^{t-j} \Pr\left(\theta_s^t\right) \left[m\left(\theta_s^t\right) - l\left(\theta_s^{t-1}\right)\right]$$

subject to

$(BC), (IC)$ *and* (PC).

◂ Recursive version

Parametrization

- Let $S = 3$, with $\bar{\theta}_1 < \bar{\theta}_2 < \bar{\theta}_3$ and let $N = 2$ with

$$\theta_{sH}^{1-\gamma} = \overline{\theta_s} + \sigma_s \frac{\sqrt{\pi_L \pi_H}}{\pi_H}, \quad \theta_{sL}^{1-\gamma} = \overline{\theta_s} - \sigma_s \frac{\sqrt{\pi_L \pi_H}}{\pi_L}$$

  implying $\mathbb{E}\left[\theta_s^{1-\gamma}\right] = \overline{\theta_s}$ and $std\left(\theta_s^{1-\gamma}\right) = \frac{\sigma_s}{\sqrt{\pi_L \pi_H}}$.

- Preferences of the workers:

$$U(c, l) = \frac{1}{1-\rho}\left(c - \psi \frac{h^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right)^{1-\rho}$$

- Preferences of the entrepreneurs:

$$U(c) = \frac{c^{1-\rho}}{1-\rho}$$

- Technology: $f(k, n) = Ak^{\alpha} n^{\eta}$, where $\gamma = \alpha + \eta$.

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